Mathematics II

(English course)

Second semester, 2012/2013

Exercises (6)

1. Among the following functions, find those which are homogeneous or positively homogeneous and report their degree of homogeneity.

   (a) $f(x,y) = \ln \frac{xy^3}{x^4+y^4}$;

   (b) $f(x,y) = \ln \frac{xy^3}{x^2+y^2}$;

   (c) $f(x,y,z) = \frac{x^2+y^2}{z^2y^2}$;

   (d) $f(x,y) = \begin{cases} (x-y)e^{\frac{xy}{x^2+5y^2}}, & \text{for } (x,y) \neq (0,0), \\ 0, & \text{for } (x,y) = (0,0); \end{cases}$

   (e) $f(x,y,z) = xy^2 + x^\alpha z^{1+3\beta} + x^{1-\alpha} y^{\frac{\alpha}{4}} z^{\alpha+\beta-2}$

2. Use Euler's equality to find the values of the parameters α, β that make the following functions positively homogeneous.

   (a) $f(x,y,z) = \frac{\sqrt{xz}}{x^2+y^2} + x^{2\alpha+\beta} z + x^{\alpha+\beta} y^{2+\beta}$;

   (b) $f(x,y) = \frac{x^{2+3\alpha} y^\alpha + x^{\alpha-\beta}}{y^\beta}$.

3. Consider a function $f : \mathbb{R}^n \setminus \{0\} \mapsto \mathbb{R}$, homogeneous of degree α.

   Show that the the following assertions are true:

   (a) If $\alpha > 0$ and f is continuous, then $\lim_{x \to 0} f(x) = 0$.

   (b) If $\alpha < 0$ and f is not identically zero, then $f(x)$ does not converge towards any finite number when $x \to 0$.

   (c) If $\alpha = 0$, then $\lim_{x \to 0} f(x)$ exists if and only if f is constant.